Understanding Temperaments

The purpose of this short text is to give the reader a basic understanding of the various temperaments and tunings used on keyboard instruments (harpsichord, organ) in the past. It will not give detailed tuning instructions (my next project) nor much more than general indications on the suitability of the various historical temperaments in different contexts.

When discussing temperaments, one cannot avoid being a bit technical. However, I have also tried to be practical by discussing temperaments that can be useful to modern keyboardists, and by stressing their important acoustical properties (i.e. how they sound) rather then getting into some complex theories. No special skills in mathematics are required of the reader (footnotes will be used to convey some extra material).

This text is complemented by a Java applet that demonstrates tunings and temperaments (I'm also thinking of a virtual instrument one could actually practice tuning on), and by some short musical examples in various tunings and temperaments (MIDI and some .au files). A PDF version of this document (about 65K -- you'll need Adobe's Acrobat reader to read it) is also available for printing (will give you much better graphics).

Anyone seriously interested in temperaments **must** read **Margo Schulter**'s remarkable Pythagorean Tuning and Medieval Polyphony. Notwithstanding the title, her discussion of Baroque meantones and irregular temperaments is also very thorough. In addition to the physics of the problem, she also addresses the musical and musicological implications, with many references to the sources of the time.

 Version 1.2
(incorporates a few changes inspired by Margo Schulter's article).

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1. Introduction

1.1 The problem

The need for temperament arises because it is impossible to have octaves, fifths, thirds, etc., all pure at once, or, in other words, because the ratios of the different pure intervals are incompatible. Two important examples will demonstrate this. Suppose we start on C and tune up 12 pure fifth: we arrive at B# which, on our keyboards, is the same key as C (we will not consider here keyboards with more than 12 notes per octave). However that B# is not in tune, as an octave, with the initial C. This can be verified by experiment (tuning), by simple arithmetic (multiplying ratios), or using cents as we will see later. Similarly, if, starting from C again, we tune four fifths up to E, the interval C-E thus formed is much wider than a pure major third.

To have false octaves has always been unthinkable, so we will have to accept that some of the other intervals will be more or less out of tune, often deliberately so as when we **temper** an interval (i.e. tune it slightly false in the process of setting a temperament). We will see later what compromises can be made.

1.2 Cents

To study the problem in more detail, we will need a unit in which to express the sizes of intervals. One very convenient unit (which we will use) is the **cent** (see footnote 1) which is defined as the one hundredth part of the equal tempered semitone we are all familiar with (or the twelve hundredth part of an octave).

From acoustics, we know that **pure intervals** correspond to simple ratios (of the frequencies involved) such as, for example, 3/2 for the fifth. Table 1 gives the size in cents of the pure consonant intervals, computed (see footnote 2) from the given ratios. Sizes in cents (as with semitones) can be added or subtracted as we add or subtract the corresponding intervals. For example, the major second obtained by tuning a pure fifth up then a pure fourth down is 702 - 498 = 204 cents (or 7 - 5 = 2 semitones, ration 9/8).

interval	ratio	size cents
octave	2/1	1200
fifth	3/2	702
fourth	4/3	498
major third	5/4	386
minor sixth	8/5	814
minor third	6/5	316
major sixth	5/3	884
major second	9/8	204

Table 1. Consonant intervals

We can now express the examples above (section 1.1) in cents. If, from C, we tune 12 pure fifths up, we will form an interval of 12 × 702 = 8424 cents. On our keyboards, this corresponds to 7 octaves or 7 × 1200 = 8400 cents. The difference between the two is 24 cents and is known as the **ditonic comma**. Similarly, four pure fifths up from C give 4 × 702 = 2808 cents. Subtracting two octaves (2 × 1200 = 2400), we find that the third thus formed is 408 cents, 22 cents larger than a pure third. This difference is known as the **syntonic comma**, and this wide third is known as the **Pythagorean third** and sounds quite harsh (or tense, depending on the point of view).

1.3 Solutions

In tuning the 12 fifths up from C, we have gone around the circle of fifths as we think of it today where enharmonics are equivalent (and we shall use it in this sense here even though it does not strictly apply to temperaments that have no enharmonics and hence do not allow complete freedom of modulation). To close the circle, we must of course make B# equal to C, and we will have to, in some way, distribute the ditonic comma by flattening some or all of the fifths until the total amount of flattening around the circle is 24 cents. We will say that the sum of the **deviations** (from pure, not equal temperament) of all the fifths must add up to **-24** cents (negative since the B# was initially too high). The various tunings and temperaments are then simply various solutions to the problem of dispersing, consciously or not, this comma.

A **tuning** is laid out with nothing but pure intervals, leaving the comma to fall as it must. A **temperament** involves deliberately mistuning some intervals to obtain a distribution of the comma that will lead to a more useful result in a given context. Solutions can be grouped into three main classes:

1. Tunings (Pythagore, just intonation),
2. Regular temperaments where all fifths but the wolf fifth are tempered the same way, and
3. Irregular temperaments where the quality of the fifths around the circle changes, generally so as to make the more common keys more consonant.

Temperaments are further classified as **circulating** or **closed** if they allow unlimited modulation, i.e. enharmonics are usable (equal temperament, most irregular temperaments), **non-circulating** or **open** otherwise (tunings, most regular temperaments).

The choice of a particular solution depends on many factors such as

- the needs of the music (harmonic vs melodic, modulations),
- the tastes of the musicians and hearers,
- the instrument to be tuned (organ vs harpsichord -- tuning the former is much more work so one needs a more versatile solution),
- aesthetics (Gothic's tense thirds and pure fifths vs the stable, pure thirds of the Renaissance and Baroque) and theoretical considerations, and
- ease of tuning (equal temperament is one of the more difficult).

Except for very special circumstances, I think one need only master a few tunings/temperaments, for example (in approximate order of difficulty) Pythagore's tuning, Aaron's meantone, Kirnberger, and, inevitably, equal temperament.

We will look in more detail at some of the more important solutions to this problem after some further preparation.

1.4 Circle of fifths

I find that the easiest way to visualize and understand a particular temperament is with the aid of a diagram representing the **circle of fifths** on which one can describe the temperament schematically showing how the -24 cents needed to close the circle are distributed, i.e. by how much each fifth is tempered or out of tune. Once the size of each fifth is fixed, the sizes of all the other intervals are necessarily also fixed. We will be interested in the deviations (differences) from pure of the various consonant intervals in order to find out how a particular temperament will sound. Note that the fourth and sixths, being inversions of the fifth and thirds, will have the same deviations as the latter but in the opposite direction. For example, if a major third is 14 cents sharp, then the corresponding minor sixth will be 14 cents flat.

To compute the size of the other intervals, we will consider them as being "formed" of those fifths (or fourths if going counterclockwise) which separate the two notes of the interval on the circle. We will use the shortest route to simplify the computations, but the other way around would give the same results. This does not, in general, correspond to the actual process of tuning such as, in particular, when one of the fifths involved is a wolf fifth.

Figure 1 shows how the major and minor thirds are considered to be "formed" in terms of fifths or fourths.

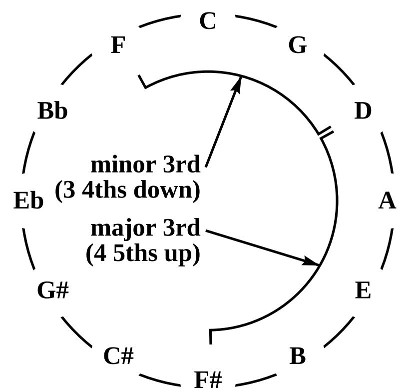


Figure 1. Forming intervals

We have already seen that four pure fifths give a Pythagorean major third of 408 cents. If some or all of the fifths forming (contained in) a given major third are tempered, we obtain the size of the major third by simply adding the deviations of the fifths (which will be negative if they are flattened as is normally the case) to 408 cents. For example, if all four fifths of a major third are tempered by -2 cents, the major third will be 408 + 4 × -2 = 400 cents (the equal-tempered third).

A minor third is formed with three ascending fourths. If the fourths are pure, the minor third will be 3 × 498 = 1494 cents; subtracting an octave we find that the minor third thus formed is 294 cents, 22 cents flat, and is the **Pythagorean minor third**. And, as above, if the fourths are tempered, we add their deviations (which will be opposite to those of the corresponding fifths) to 294 cents to obtain the size of the resulting third. The sizes of other intervals can be obtained similarly.

As a consonant interval deviates more and more from pure, it eventually becomes a **wolf** interval, i.e. too false to be musically useful. The point at which this happens depends on what hearers are used to and are willing to tolerate. For thirds, we usually take the deviation of the Pythagorean third as the limit, i.e. about 22 cents. For fifths, half a syntonic comma, i.e. 11 cents, is about the limit. These numbers are derived from what is found in old temperaments, i.e. what appears to have been accepted at some time. That is not to say that modern ears will accept those limits.

Dissonant intervals will not be considered much here since it seems to matter little whether a dissonant interval is in tune or not. Nevertheless, dissonant intervals can sound different in the different temperaments: one that strikes me in particular is the tritone of Aaron's meantone (more on this later).

2. Equal temperament

Because of its complete symmetry and complete freedom to modulate, the most obvious solution today, and the one almost universally adopted, is of course **equal temperament** in which the comma is evenly distributed around the circle of fifths: each fifth is thus flattened by 2 cents for a total of 12 × -2 = -24 cents), and is 702 - 2 = 700 cents wide. The major thirds are 408 + 4 × -2 = 400 cents, 14 cents sharp. The minor thirds are 294 + 3 × 2 = 300 cents, 16 cents flat. These intervals come out as multiples of 100 cents as was to expected from the definition of the cent (above). In equal temperament, all keys are identical except for pitch. The fifths are nearly pure, and the thirds fairly heavily tempered.

However, equal temperament has not been the obvious solution in all contexts: the complete freedom to modulate has not always been necessary, and the sameness of all keys found in equal temperament has not always been appreciated. In addition, equal temperament is one of the more difficult temperaments to tune. Equal temperament was used quite early on fretted instruments (it's the only arrangement that works, because many strings share a fret).

3. Pythagore's tuning

Historically, the first important system is **Pythagore**'s tuning (which is not a temperament as no interval is tempered). It is obtained by tuning a series of 11 pure fifths, typically from Eb to G#, the remaining fifth (diminished sixth really) receiving all of the ditonic comma and therefore being 24 cents flat. The resulting diagram is shown in Figure 2. All thirds, major or minor, except those which include the diminished sixth, will be Pythagorean thirds since they include only pure fifths, and will therefore be quite tense (harsh). The four major thirds (diminished fourths) which include the diminished sixth will be 408 - 24 = 384 cents, nearly pure (2 cents flat). Similarly for the minor thirds. In brief, except for one wolf fifth, all intervals are usable if not pleasant. In the common keys, the thirds will be harsh which makes this tuning unsatisfactory for tonal music; but it can be quite effective for medieval music where in fact the tenseness of the thirds was musically important. Around 1400, the four nearly pure thirds were put to good use, contrasting with the usual tense thirds, by placing the wolf between B and Gb; triads such as D-F#-A became nearly just. The sharp keys now moved to the new Renaissance ideal (stable thirds), while the flat keys stayed with the old ideal. Also, the semitones are of two different sizes (90 and 114 cents) which lends a characteristic expressiveness to this tuning. Pythagore's tuning was prevalent in most of the Gothic era.

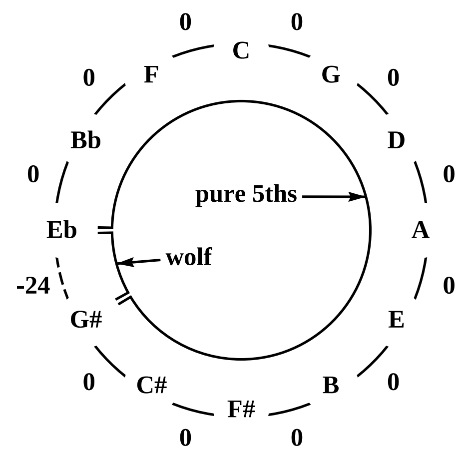


Figure 2. Pythagore's tuning

It is ironic that the most modern temperament, equal temperament, is in fact quite close to Pythagore's tuning with its nearly pure fifths and fairly tense thirds, and it is therefore quite effective for Medieval music.

4. Just intonation

We will now take a look at just intonation. **Just intonation** is based only on pure octaves, fifths and thirds, i.e. simple-ratio intervals: any note can be obtained from any other by tuning pure fifths and/or thirds. This tuning is mostly of theoretical interest since any attempt to impose it upon fixed-intonation instruments necessarily leads to serious flaws which make it impractical. As Barbour said in *Tuning and Temperament*, "it is significant that the great music theorists ... presented just intonation as the theoretical basis of the scale, but temperament as a necessity".

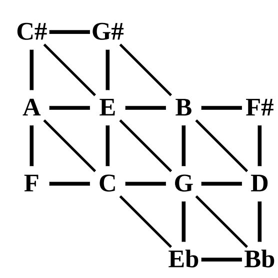


Figure 3. Just intonation

We will look at **Marpurg**'s monochord number 1 which Barbour presented as the model form of just intonation. Figure 3 shows how the various notes are obtained from one another by tuning pure intervals: horizontal lines represent pure fifths, vertical lines pure major thirds, and diagonal lines pure minor thirds. For example, B is obtained from C by tuning a fifth to G, then a third.

This results in the circle shown in Figure 4. The various segments of pure fifths are liked by pure thirds. Notice that, besides the diminished sixth, there are three bad fifths which were necessary to obtain the pure thirds (and the deviations of the fifths correspond to the difference between a pure and a pythagorean third). In particular, there is always one bad fifth between C and E, typically D-A (as here), or G-D, which is a serious flaw! Any triad whose notes are neighbourghs in Figure 3 will be pure, e.g. F-A-C or C-Eb-G, but others, such as D-F#-A will be unusable.

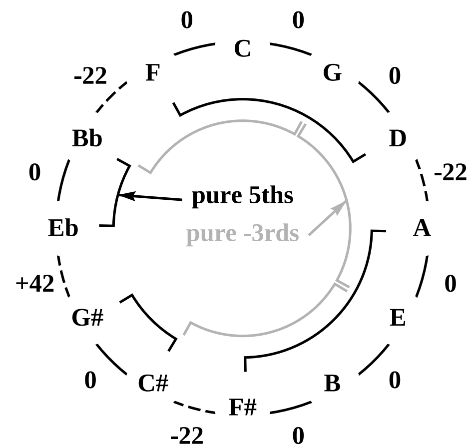


Figure 4. Just intonation

5. Regular temperaments

Regular temperaments are characterized by having all fifths except the wolf tempered by the same amount. So the different keys sound identical as long as no interval involves the wolf fifth. There is a whole range of such temperaments, characterized by how much the good fifths are tempered.

5.1 Aaron’s meantone

A very important solution, from a practical point of view, is **Aaron**’s meantone temperament. It is a practical compromise on just intonation. In this temperament, all the good fifths are tempered sufficiently so that a succession of four of them produces a pure major third which had become musically important. To achieve this, the total deviation of the four fifths will have to be -22 cents, hence -5.5 cents per fifth (or 1/4 of a comma, hence the name "1/4 comma meantone" -- meantone because each third is divided into two equal whole tones).

This results in the circle shown in Figure 5 (in just intonation, one in every four fifths was 22 cents flat, a whole comma -- compare Figures 4 and 5). The total deviation of 11 such fifths is 11 × -5.5 = -60.5 cents (please bear with the fractional cents; this excessive precision is maintained only to make the major thirds exactly pure). Hence, the remaining wolf fifth (diminished sixth) will have to be 36.5 cents sharp to bring the total deviation around the circle to -24 cents. This wolf fifth is conventionally placed between G# and Eb, but is frequently placed elsewhere, depending on the music to be played (on the harpsichord, a few notes can easily be retuned between pieces). The major thirds that do not include the diminished sixth are pure by design. Those that include it (they are, in fact, diminished fourths) are 408 + 3 × -5.5 + 36.5 = 428 cents (42 cents sharp) and are not usable as major thirds. Similarly with the minor thirds. In meantone, only 16 out of 24 possible major and minor triads are usable which severely restricts modulation (notes cannot be used enharmonically, e.g. a G# will not do where and Ab is wanted). However, the good triads sound more harmonious than in equal temperament because of the pure thirds, even though the fifths are tempered nearly three times as much; this makes

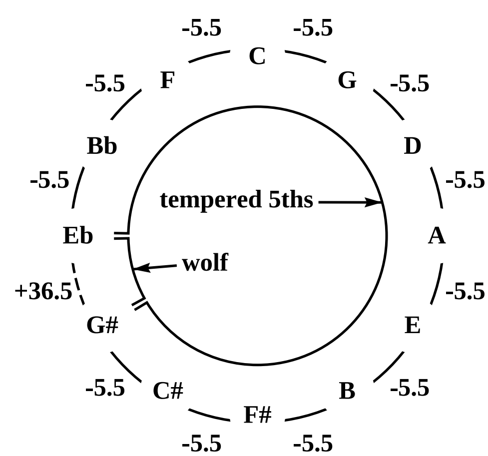


Figure 5. Aaron’s meantone

meantone interesting for music which does not modulate beyond its bounds (or does so intentionally).

In a way, this is a worse solution than Pythagore's tuning: it has more wolves, and the wolf fifth is much worse; this was the price to pay to get the stable thirds.

The diminished fourth F#-Bb (enharmonically equivalent to a major third in equal temperament) is a wolf in meantone when trying to use it as a third, but it is usable in a context where it is intended such as in the *tremblement appuyé* on A found in the last measures of the Sarabande from d'Anglebert shown in Figure 6 (from the second *suite* in G minor). Of course, such a sequence will not sound the same as in equal temperament (.au files demonstrating this: in equal temperament, in Aaron's meantone).

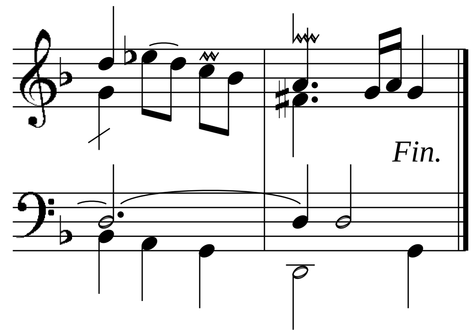


Figure 6. Sarabande de d'Anglebert

Let us, as an aside, compute the size of the tritone F-B. It is formed of six fifths. If they were pure, the size of the tritone would be 6 × 702 cents; each fifth being 5.5 cents flat, the size is therefore 6 × (702 - 5.5) = 4179 cents. Subtracting 3 octaves, we get 4179 - 3600 = 579 cents. This happens to be close to the size of a pure interval whose ratio is 7/5, simple enough to be perceived, and which corresponds to 583 cents. This explains why it sounds different from the tritone of equal temperament (600 cents).

5.2 Silbermann and others

Temperaments in which the good fifths (11) are all the same size (except for the wolf), such as Aaron's meantone we have just seen, are called regular. There is a whole range of such temperaments starting with fifths tempered yet more heavily (called **negative** meantones since the major thirds are smaller than pure) thru Aaron's temperament, equal temperament, to, finally, Pythagore's tuning with the good fifths being pure. Typical within this range is **Silbermann**'s temperament (used in the high Baroque for organs) in which the fifths are tempered by 1/6 of a syntonic comma or nearly 4 cents. An analysis similar to the above will show that the good major thirds are about 7 cents sharp and the good minor thirds about 11 cents flat, and that the wolves are still present though a bit more quiet.

Temperament	Salinas 1/3 comma	Aaron 1/4 comma	Silbermann 1/6 comma	Equal 1/11 comma	Pythagore
fifth	-7.3	-5.5	-3.7	-2	0
dim. sixth	+56.7	+36.5	+16.3	-2	-24
major third	-7.3	0	+7.3	+14	+22
dim. fourth	+56.7	+42	+27.3	+14	-2
minor third	0	-5.5	-11	-16	-22
aug. second	-64	-47.5	+31	-16	+2
diff. enharm.	+64	+42	+20	0	-24
beats init. GE	0	3	6	9	n/a

Table 2. Regular temperaments

Table 2 summarizes the properties of some of the more important regular temperaments: the numbers represent deviations from pure in cents. The table gives the difference between enharmonic equivalents (such as G# and Ab), a positive number indicating that the sharp enharmonic (such as G#) is the lower of the two; this number is also the difference between the chromatic and diatonic semitones. The last row will be explained with the forthcoming practical tuning instructions.

The "ultimate" regular temperament is of course **equal temperament**: the wolf is gone and replaced by a fifth of the same size as all the others. But it is atypical and uncharacteristic of regular temperaments because of its rather wide thirds and the absence of any difference between enharmonics.

6. Irregular temperaments

The last group of temperaments we shall look at are the **irregular temperaments** (also know as **well** temperaments) which are now believed to have been very important in the past (especially during the Baroque). They are characterized by having more than one size of good fifths (and thus thirds), by having no wolf intervals to limit modulation (as in the previous temperaments except equal), and by having a more or less orderly progression in the acoustic quality of the triads from near to remote keys, i.e. a tonal palette. Generally speaking, the ditonic comma (-24 cents) is distributed unevenly around the circle: most of it is given to the fifths of the near keys, and little, if any, to the fifths of the remote keys (in some cases, such as the French temperament ordinaire, the first fifths are tempered a bit too much, with the result that the last fifths of the circle have to be a bit sharp, a waste). The consequence of the above arrangement is that, in the near keys, the thirds are much purer and the fifths less so than in the remote keys. In the near keys, irregular temperaments resemble meantone, and in the remote keys, they resemble (the near keys of) Pythagore's tuning (with the tense thirds). This gives added variety to modulation, which was appreciated in the past, and probably explains the different characters of the different keys mentioned in the literature of the time. This kind of variety is absent in the regular temperaments including equal temperament of course.

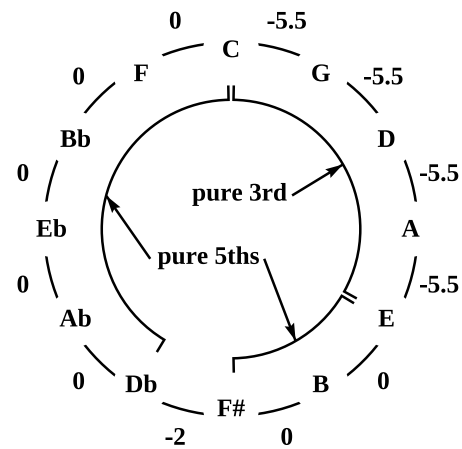


Figure 7. Kirnberger

6.1 Kirnberger

A very typical temperament within this group is **Kirnberger**'s temperament (III?) (as it was taught to me -- similar to Kirnberger's temperament in Barbour's book except that, here, the syntonic comma is distributed to 4 fifths instead of 2). One starts with a pure major third from C to E, with the four fifths within it tempered as in Aaron's meantone. With this pure major third, we have already accumulated -22 cents, nearly all that we need to close the circle (-24 cents -- the two commas being conveniently nearly equal). If we tune the remaining fifths pure, the last fifth that will close the circle will be 2 cents flat (as in equal temperament). From E, one conventionally tunes two pure fifths up to F# and, from C, five pure fifths down to Db. This gives us the circle shown in Figure 7. The major thirds vary gradually from pure (C-E) to Pythagorean (e.g. Db-F). A similar effect is observed for the minor thirds which vary from nearly pure (e.g. A-C about 5 cents flat) to Pythagorean (e.g. Bb-Db). The major triad on C is a meantone triad, and the one on Db is Pythagorean.

6.2 Vallotti

Another typical example is **Vallotti**'s temperament. Here, the comma is distributed equally to 6 consecutive fifths, those involving no raised keys, the others being pure. This gives the circle shown in Figure 8. Here again, the major thirds vary from not quite pure (6 cents sharp) to Pythagorean, and similarly for the minor thirds.

This temperament can be rotated halfway to obtain a "Well Pythagore", i.e. a temperament that is Pythagorean in its near keys, yet usable in all keys.

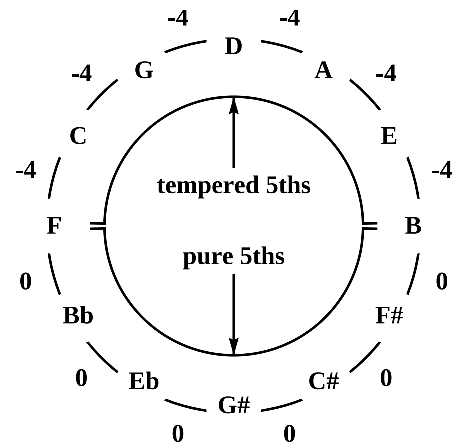


Figure 8. Vallotti

6.3 Werckmeister

Werckmeister's temperament is a bit less symmetric. The three fifths between C and A are tempered such as to give a slightly wide major sixth. The other tempered fifth is between B and F# giving the circle in Figure 9. The fact that the tempered fifths are not consecutive makes this temperament less unequal than Kirnberger's, even though the fifths are tempered essentially the same in both. The pattern of the thirds is very similar to Vallotti's temperament.

6.4 Summary

Some properties of a few important irregular temperaments are summarized in Table 3. The last line, "inequality", gives a rough measure of the variety in modulation provided by the temperament (see footnote 3).

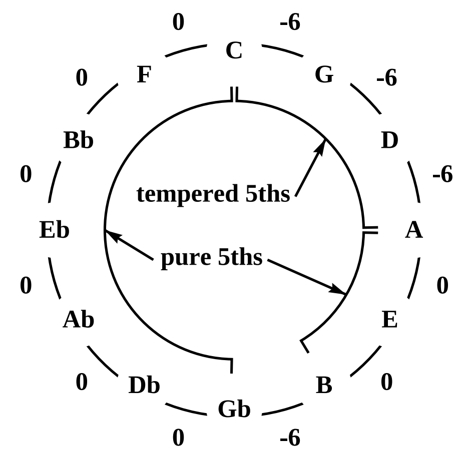


Figure 9. Werckmeister

The "ultimate" irregular temperament is of course **equal temperament** with all fifths equal. But it is atypical of irregular temperaments because it is completely regular and all keys are musically equivalent with their uniform and active thirds (and no stable thirds as in meantone). It has one color, and, from a Renaissance/Baroque point of view, the wrong color.

The inequality of equal temperament is 0.0; in practice however, the inequality of a competent tuner's work might be around 1.0 (based on data in Grove's, 1965, article on tuning). If one plays music typical of the 18th century using one of the irregular temperaments above, one will find that, on average (weighted) the thirds are about 9 cents sharp as opposed to 14 sharp, as always, in equal temperament. The thirds, therefore, sound purer, but the fifths are more tempered.

Temperament	French ordinaire	Kirnberger	Werck- meister	Vallotti	Equal temperament
no. pure 5ths	1	7	8	6	0
worst 5th	-4.5	-5.5	-6	-4	-2
best +3rd	+4	0	+4	+6	+14
worst +3rd	+29	+22	+22	+22	+14
unequality	8.0	6.2	5.3	5.3	0.0

Table 3. Irregular temperaments

Many other interesting temperaments exist, and we might close by proposing to the interested reader that he or she studies a few of them on his/her own. For example, in **Marpurg**'s I temperament, three fifths are tempered by 8 cents and placed symmetrically around the circle, the others being pure. This results in an approximation of equal temperament. In **Grammateus**' temperament, the diatonic notes are tuned according to Pythagore's tuning, and the chromatic notes are placed halfway between neighboring diatonics.

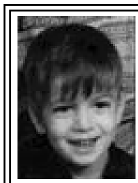
7. Other resources

- See the on-line version for other resources

Footnotes:

1. A logarithmic unit, just as the semitone.
2. The size of an interval is 1200 × log2 (f2/f1) where f1 and f2 are the frequencies of the two notes forming the interval. If the interval is pure, f2/f1 is the same as the corresponding simple ratio. Note that "log2" means "log base 2".
3. It is the mean absolute deviation of the major thirds around +14 cents, the mean.

About the author:



My main work was software development (telephony signalling protocols) at Nortel. In a previous life, I was very much into (early) music, and also into tuning (as a semi-professional harpsichord tuner); this led me to a study of historical tunings and temperaments.

Page URL: http://pages.globetrotter.net/roule/temper.htm

ANNEX: Cent values for a few temperaments

The following are my notes from 20+ years ago when I wrote the initial version of my "Understanding Temperaments" web page. Unfortunately, I didn't keep good notes as to how I arrived at these numbers (in particular, which source I used). Much probably comes from Barbour. So with all the applicable caveats...

For each tuning/temperament, the following gives, on two lines:
- The value in cents for each note on the circle of fifths; for closed temperaments, the "unequality" is given at the end of that line.
- The amount by which each fifth deviates from pure.

The following assumes values for the pythagorean (ditonic) and syntonic commas of 24 cents and 22 cents, respectively.

Notes:

- "French (2)" corresponds to Lindsay's definition of the French "tempérament ordinaire". Similar but not the same as what I had.

- Marpurg shows an unequality of zero as for equal temperament. All thirds in Marpurg are identical to equal-tempered thirds. Three out of every four fifths are pure, and one fifth takes all the tempering that, in equal temperament, is spread over four fifths.

- The "French" temperaments (last four lines) show fifths tuned wide of pure. As a result, other fifths will have to be even narrower for the sum of the deviations around the circle to add up to −24 cents. A waste. "Mean semitone" also shows the same characteristic.

- In "Silbermann" and "Salinas", .3 and .4 mean 1/3, and .6 and .7 mean 2/3. In "French (2)", .7 and .8 mean 1/4.

- Don't worry too much about accuracy. A temperament won't be completely different just because one rounds the cent values.

Thanks to John Wood for prompting and help in preparing this page (esp. for "French (2)" above).

	C	G	D	A	E	B	F#	C#	G#	Eb	Bb	F	(C)	
Equal	000.0	700.0	200.0	900.0	400.0	1100.0	600.0	100.0	800.0	300.0	1000.0	500.0		
		−2.0	−2.0	−2.0	−2.0	−2.0	−2.0	−2.0	−2.0	−2.0	−2.0	−2.0	−2.0	
Pythagore	000.0	702.0	204.0	906.0	408.0	1110.0	612.0	114.0	816.0	294.0	996.0	498.0		
		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	−24.0	0.0	0.0	0.0	
Aaron	000.0	696.5	193.0	889.5	386.0	1082.5	579.0	75.5	772.0	310.5	1007.0	503.5		
		−5.5	−5.5	−5.5	−5.5	−5.5	−5.5	−5.5	−5.5	+36.5	−5.5	−5.5	−5.5	
Silbermann	000.0	698.3	196.7	895.0	393.3	1091.7	590.0	88.3	786.7	305.0	1003.3	501.7		
		−3.7	−3.6	−3.7	−3.7	−3.6	−3.7	−3.7	−3.6	+16.3	−3.7	−3.6	−3.7	
Salinas (−ve)	000.0	694.7	189.3	884.0	378.7	1073.3	568.0	62.7	757.3	316.0	1010.7	505.3		
		−7.3	−7.4	−7.3	−7.3	−7.4	−7.3	−7.3	−7.4	+56.7	−7.3	−7.4	−7.3	
Kirnberger	000.0	696.5	193.0	889.5	386.0	1088.0	590.0	90.0	792.0	294.0	996.0	498.0		uneq=6.2
		−5.5	−5.5	−5.5	−5.5	0.0	0.0	−2.0	0.0	0.0	0.0	0.0	0.0	
Vallotti	000.0	698.0	196.0	894.0	392.0	1090.0	592.0	94.0	796.0	298.0	1000.0	502.0		uneq=5.3
		−4.0	−4.0	−4.0	−4.0	−4.0	0.0	0.0	0.0	0.0	0.0	0.0	−4.0	
Werkmeister	000.0	696.0	192.0	888.0	390.0	1092.0	588.0	90.0	792.0	294.0	996.0	498.0		uneq=5.3
		−6.0	−6.0	−6.0	0.0	0.0	−6.0	0.0	0.0	0.0	0.0	0.0	0.0	
Marpurg	000.0	702.0	204.0	906.0	400.0	1102.0	604.0	106.0	800.0	302.0	1004.0	506.0		uneq=0.0
		0.0	0.0	0.0	−8.0	0.0	0.0	0.0	−8.0	0.0	0.0	0.0	−8.0	
Mean semitone	000.0	696.5	193.0	889.5	386.0	1082.5	600.0	96.5	793.0	289.5	986.0	503.5		uneq=9.3
		−5.5	−5.5	−5.5	−5.5	−5.5	+15.5	−5.5	−5.5	−5.5	−5.5	+15.5	−5.5	
Grammateus	000.0	702.0	204.0	906.0	408.0	1110.0	600.0	102.0	804.0	306.0	1008.0	498.0		uneq=5.3
		0.0	0.0	0.0	0.0	0.0	−12.0	0.0	0.0	0.0	0.0	−12.0	0.0	
French	000.0	697.5	195.0	892.5	390.0	1087.5	587.0	87.0	789.0	294.0	998.5	502.5		uneq=8.0
		−4.5	−4.5	−4.5	−4.5	−4.5	−2.5	−2.0	0.0	+3.0	+2.5	+2.0	−4.5	
French (2)	000.0	696.5	193.0	889.5	386.0	1082.5	581.8	81.0	783.0	289.5	996.5	503.5		uneq=11.4
		−5.5	−5.5	−5.5	−5.5	−5.5	−2.7	−2.8	0.0	+4.5	+5.0	+5.0	−5.5	
Rameau	000.0	696.5	193.0	889.5	386.0	1082.5	584.5	86.5	788.5	298.0	1007.0	503.5		uneq=11.5
		−5.5	−5.5	−5.5	−5.5	−5.5	0.0	0.0	0.0	+7.5	+7.0	−5.5	−5.5	
Irr Fr 17e	000.0	697.0	194.0	891.0	388.0	1085.0	582.0	79.0	776.0	292.0	998.0	503.0		uneq=12.7
		−5.0	−5.0	−5.0	−5.0	−5.0	−5.0	−5.0	−5.0	+14.0	+4.0	+3.0	−5.0	
	C	G	D	A	E	B	F#	C#	G#	Eb	Bb	F	(C)	